Commun. math. Phys. 26, 87—101 (1972)

Solutions of the Einstein-Maxwell Equations with Many Black Holes

J. B. HARTLE* and S. W. HAWKING
Institute of Theoretical Astronomy, Cambridge, England

Received January 10, 1972

Abstract. In Newtonian gravitational theory a system of point charged particles can be arranged in static equilibrium under their mutual gravitational and electrostatic forces provided that for each particle the charge, e, is related to the mass, m, by $e = G^{\frac{1}{2}}m$. Corresponding static solutions of the coupled source free Einstein-Maxwell equations have been given by Majumdar and Papapetrou. We show that these solutions can be analytically extended and interpreted as a system of charged black holes in equilibrium under their gravitational and electrical forces.

We also analyse some of stationary solutions of the Einstein-Maxwell equations discovered by Israel and Wilson. If space is asymptotically Euclidean we find that all of these solutions have naked singularities.

I. Introduction

In Newtonian theory a system of point charged particles can remain in static equilibrium if the charges e_i are all of the same sign and related to the masses m_i by

$$|e_i| = G^{\frac{1}{2}} m_i . \qquad (1.1)$$

No matter how the particles are arranged, if this condition is satisfied, the electrostatic repulsions exactly balance the gravitational attractions. In 1947, Majumdar [1] and Papapetrou [2] independently discovered a class of static solutions to the source free Einstein-Maxwell equations which correspond to this Newtonian situation. The source free solutions given by Majumdar and Papapetrou are not geodesically complete. One way of completing them is to match the solutions to static interior solutions of dust whose charge density equals its mass density [3]. It is even more interesting, however, to study the analytic extension of the source free solutions themselves in view of the fruitful studies already carried out on the analytic extensions of the Schwarzschild [4], Reissner-Nordstrom [5] and Kerr [6–7] geometries. These latter solutions are all found to be asymptotically Euclidean. They each contain (for certain

* Alfred P. Sloan Research Fellow, supported in part by the National Science Foundation.