

An Improved Formulation of Some Theorems and Axioms in the Axiomatic Foundation of the Hilbert Space Structure of Quantum Mechanics

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Abstract. In Ref. [1] the axiomatic foundation of the Hilbert space structure of quantum mechanics was outlined. Apart from a set of physically plausible axioms, the (mathematical) assumption (V 1) of the minimal-decomposition property of the basenorm space B was incorporated into the axiomatic scheme of the theory.

It is shown in the present paper that the assumption (V 1) is superfluous. In the first part of the paper we give a short summary of the axioms; in the second part the main theorems are proved without using assumption (V 1).

Introduction

In [1] a set of axioms for quantum mechanics were motivated by formulating the principal laws of measurement; from these axioms it is possible to deduce the Hilbert space structure of quantum mechanics. In [2] we have attempted to give a more detailed physical interpretation of one of these axioms. A survey of the conceptual structure of our axiomatic scheme is given in [3]. In the present paper we will propose an improved formulation of some of the axioms and theorems of [1]. The concepts and symbols as defined in [1] (Chapter III) are used throughout.

In Ref. [4] (§ 3.3) the mapping principle 3c (Abbildungsprinzip 3c) of [1], Chapter III, § 1.5, has been reformulated; it seemed more suitable to take, for the elements of \underline{K} , more general subsets of \tilde{K} than those given by the mapping principle 3c. As explained in [4], § 3.3, this may be done if the mapping principle 3c is replaced by a new set of postulates introducing a set ϕ of “selection-procedures” (Auswahlverfahren) (cf. Ref. [4], § 3.3). The mapping principle 3d (Abbildungsprinzip 3d) of Ref. [1], Chapter III, § 1.5, can also be replaced by selection-procedures in a quite