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Lorentz Cobordism

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Abstract. A Lorentz cobordism between two (in general nondiffeomorphic) 3-manifolds M_0, M_1 is a pair (M, v), where M is a differentiable 4-manifold and v is a differentiable vector field on M, such that 1) the boundary of M is the disjoint union of M_0 and M_1, 2) v is everywhere nonzero, 3) v is interior normal on M_0 and exterior normal on M_1. Such a manifold M admits a Lorentz tensor with respect to which M_0 and M_1 are spacelike hypersurfaces; thus a Lorentz cobordism is a model of a portion of a spacetime in which "the topology of spacelike hypersurfaces is changing". We discuss the form that these changes can take, and give two methods for constructing a Lorentz cobordism between two nondiffeomorphic 3-manifolds. We comment on the possible relevance of Lorentz cobordism to the problem of gravitational collapse.

I. Introduction

Suppose we cut a "slab" out of a spacetime manifold by slicing along two disjoint spacelike hypersurfaces: we are left with a 4-manifold M with boundary $M_0 \cup M_1$, where M_0 and M_1 are disjoint 3-manifolds which are spacelike hypersurfaces with respect to the Lorentz structure[1] induced on M by that on our original spacetime. If we do this to the model spacetimes that have traditionally been studied in general relativity theory (except some of those with "singularities"?), the 3-manifolds M_0 and M_1 will be diffeomorphic; there will be no "change of topology". We consider here the case in which M_0 and M_1 are *not* diffeomorphic. We shall be interested in such things as the topology of the manifold M, the causal properties of Lorentz structures on it, the singularities of Lorentz structures on it, and, ultimately, the solution of Einstein's field equations on such an underlying manifold.

We shall, however, do things the other way round from the "cutting" operation imagined above. Rather, we shall start with two non-diffeomorphic 3-manifolds M_0, M_1, and then consider how we can construct a 4-manifold M and a (singular or nonsingular) Lorentz tensor g on M such that the boundary of M is the disjoint union of M_0 and M_1, with M_0 and M_1 spacelike with respect to g.

[1] We shall use the terms *Lorentz structure* and *Lorentz tensor* synonymously to mean a globally defined, second rank, symmetric tensor of Lorentz signature $(+, +, +, -)$.