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Bogoliubov Inequalities for Infinite Systems*

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Abstract. The Bogoliubov inequalities are derived for the infinite volume states describing the thermodynamic limits of physical systems. The only property of the states required is that they satisfy the Kubo-Martin-Schwinger boundary condition.

I. Introduction

In the conventional Green's function approach to statistical mechanics all relations are first derived for strictly finite systems; the thermodynamic limit is taken at the end of the calculation. Since the original derivation [1] of the Bogoliubov inequalities was carried out within this framework, the subsequent applications had to follow the same prescription. In this way the inequalities have been applied by Josephson [2] to derive rigorous inequalities for the specific heat and by a number of authors [3–7] to show the impossibility of various kinds of long-range order in one- and two-dimensional systems. In the latter class of problems, a special difficulty arises from the fact that finite systems do not exhibit the broken symmetries usually associated with long-range order. This has led to the use of Bogoliubov's quasi-averaging method in which the finite-system Hamiltonian is modified by the addition of a symmetry breaking term, which is set equal to zero only after the passage to the thermodynamic limit. This method has never been shown to be equivalent to the more rigorous treatment of broken symmetries provided by the theory of integral decompositions of states on C^*-algebras [8, 9]; furthermore, for some problems (e.g. Bose condensation and anti-ferromagnetism) the symmetry breaking term has no physical significance.

The purpose of this paper is to show how these difficulties can be avoided by establishing the Bogoliubov inequalities directly in the thermodynamic limit. An application of the inequalities to long-range order in one- and two-dimensional systems will be published separately.

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