Commun. math. Phys. 25, 308—335 (1972)


# Dispersion Relations for the Vertex Function from Local Commutativity

## II. Two-Dimensional Dispersion Relations

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Received June 8, 1971

**Abstract.** Representation formulas for the vertex function are derived which are valid for arbitrary complex values of two of the scalar variables inside the corresponding domain of holomorphy, while the third variable is evaluated in a neighbourhood of positive real values (the corresponding physical-region singularities). The analyticity domains always include at least the corresponding "axiomatic" Källén-Wightman domain.

Two alternative versions are given. In the first formula the occurring weight functions are the boundary values of the vertex function along the real axes of the scalar variables. The values of the arguments are such that the boundary values can be expressed in terms of the causal respectively time-ordered functions, which in a well-known way via reduction technique are related to the different form factors of the underlying field theory. In the second formula the weight functions are just the on-mass-shell matrix elements of the field operators which completely describe the dynamics of the theory and e.g. contain explicit information (in their support properties) on the mass spectrum of the field theory.

The assumptions behind the results are the very general physical assumptions of Källén and Wightman, i.e. essentially Lorentz-covariance, translation-invariance, "reasonable" mass- and energy-spectrum, and local commutativity. We further need some moderate integrability- and boundedness-properties of the vertex function. It is shown that the formalism can be extended to cover all cases of at most polynomial increase of the vertex function in asymptotic directions inside the Källén-Wightman holomorphy domain.

The kernel functions in the representation formulas can be explicitly seen to result in possible singularities only along the physical-region-cuts, i.e. from the contributions of the corresponding physical states, as well as along the more complicated Källén-Wightman boundary-surfaces of the holomorphy domain, which correspond to the so-called "anomalous cuts".

The kernel functions in the second version above are closely connected to simple perturbation theoretical functions.

## 1. Introduction

The notion of causality, in more or less sophisticated mathematical formulation, has had far-reaching implications on elementary particle physics.

Thus assumptions of causality, combined with some further physical hypotheses and formulated in field theoretical language, imply analyticity