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Dispersion Relations for the Vertex Function from Local Commutativity

I. One-Dimensional Dispersion Relations

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Abstract. Dispersion relations for the vertex function are derived which are valid when two of the scalar variables are arbitrary complex inside certain domains of the product of the complex planes and the third scalar variable is evaluated just below or just above the physical region-cut.

The domains of validity of the dispersion relations for the complex variables are domains with three real dimensions and can be described as neighbourhoods of the boundaries of the "axiomatic" analyticity region of Källén and Wightman.

The discontinuity of the vertex function across the cut-surface in the third variable for such values of the remaining variables is expressed only in terms of the dynamical on-mass-shell matrix elements of the locally commuting field operators.

1. Introduction

In a series of earlier papers, hereafter called I [1], II [2], and III [3], we have derived a set of relations of the kind usually called sum rules for the vertex function both in momentum space [2, 3] and in coordinate-space [1].

The basic assumptions behind the results are the analyticity properties proved by Källén and Wightman [4] from some very general assumptions, which ought to be fulfilled in all "interesting" field theories. These authors assumed essentially that the field theory should admit

(i) *covariance* under *Lorentz transformations* and *translations*, i.e., among other things the existence of energy-momentum-operators

(ii) "*reasonable*" *mass spectrum* of the energy-momentum-operators, i.e., that the occurring energies and masses should be positive, and that the Hilbert space is spanned by eigenstates of these operators

(iii) *causality* in the form of *local commutativity* properties of the occurring field operators.