

Unitary Implementation of Automorphism Groups on von Neumann Algebras*

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Received October 2, 1971

Abstract. A necessary and sufficient continuity condition is obtained in order that a topological group of automorphisms of a semi-finite von Neumann algebra in standard form is unitarily implemented. The methods used are extended to the study of unitary implementation for a general von Neumann algebra of those automorphism groups that commute with the one-parameter modular automorphism group.

1. Introduction

The Hilbert space H of a semi-finite von Neumann algebra \mathfrak{A} in standard form can be viewed as the completion of a certain two-sided ideal $\mathfrak{m}^{1/2}$ of \mathfrak{A} . It is then not surprising that an automorphism (i.e. a $*$ -automorphism) γ of \mathfrak{A} will be implemented by a unitary operator U on H in the sense that $\gamma(A) = UAU^{-1}$ for every A in \mathfrak{A} . If Γ is a group of automorphisms of \mathfrak{A} , one might conjecture that there is a homomorphism U of Γ into the group of unitary operators on H so that the homomorphism $\gamma \rightarrow U_\gamma$ implements the action of Γ in the sense that $\gamma(A) = U_\gamma AU_\gamma^{-1}$ for every $A \in \mathfrak{A}$ and $\gamma \in \Gamma$. If Γ is a topological group and if the action of Γ is continuous in the sense that for every fixed $A \in \mathfrak{A}$ $\gamma \rightarrow \gamma_0$ in Γ implies $\gamma(A) \rightarrow \gamma_0(A)$ in the weak operator topology of \mathfrak{A} , then one wishes U to be continuous in the sense that $U_\gamma \rightarrow U_{\gamma_0}$ in the strong (equivalently, weak) operator topology of H whenever $\gamma \rightarrow \gamma_0$.

We shall show that every continuous automorphism group of semi-finite von Neumann algebra in standard form on the Hilbert space H is implemented by a (continuous) unitary representation on H , provided a certain joint continuity condition holds. We show that every continuous locally compact automorphism group automatically has this joint continuity property by showing that a continuous locally compact automorphism group of an arbitrary von Neumann algebra is unitarily implemented on some Hilbert space (depending presumably on the group) on which the algebra is faithfully represented as a von Neumann algebra.

* This research was partially supported by the National Science Foundation.