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# Surface Tension in the Ising Model

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**Abstract.** We investigate the problem of a microscopic definition of the surface of separation between two phases in the special case of the 2-dimensional Ising model. We show how this leads to a definition of the surface tension which appears, in this context, as the logarithm of a partition function over a set of random surfaces. We also discuss the more general problem of defining the surface tension in an Ising ferromagnet with arbitrarily extended attractive interaction.

## 1. Introduction

The problem of giving a statistical-mechanical definition of surface tension does not seem to have been even posed in a completely satisfactory way mainly as a consequence of our inability to give precise meaning to the surface of separation of two pure phases and, even worse, to the very concept of coexisting phases. It is well known that we even lack a proof of the existence of a phase transition for a continuous system. However, for lattice systems one can rigorously show that phase-transitions occur under certain conditions, and the phenomenon of phase separation has recently been so deeply investigated that, as we are going to show, it is much more hopeful to investigate the problems connected with the surface tension in these systems.

In fact in a fundamental paper Minlos and Sinai [7, 8], hereafter referred as MS, have considered a $\nu$-dimensional Ising ferromagnet enclosed in a box $\Omega$ surrounded by a layer of spins up and with a fixed total magnetization:

$$M = (\alpha m^* + (1-\alpha)(-m^*))\,|\Omega| \qquad 0<\alpha<1 \tag{1.1}$$

where $m^*$ is the spontaneous magnetization. They have proved that if the temperature is very low and one picks up at random a configuration of spins out of the canonical ensemble defined by fixing the total magnetization as in (1.1), then with very large probability (tending to 1 as $|\Omega| \to \infty$) this configuration will consist of a "drop", roughly square in shape, with