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The Equation Curl $W_\mu(x) = 0$ in Quantum Field Theory*

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Abstract. In one time and arbitrarily many space dimensions we obtain necessary and sufficient conditions for the existence of a local operator solution of the equation $\partial_\nu \omega = W_\nu$. Here the given local fields W_ν satisfy $\partial_\mu W_\nu - \partial_\nu W_\mu = 0$ and the spectrum of the two point function $(\Omega, W_\mu(x)\, W_\nu(y)\Omega)$ is assumed to have a mass gap.

For the mathematical treatment of local field theoretic models involving pseudovector or vector fields the curl of which vanishes[1] it is important to know whether these fields are gradients of local, pseudoscalar or scalar primitive fields respectively and whether the primitive fields are relatively local to the rest of the underlying fields of the model. For the pseudoscalar current: $\tilde{\psi}\gamma^5\gamma^\mu\psi$: where ψ denotes a free massive Dirac field in one time and one space dimension the primitive field can be expressed in terms of creation and annihilation operators and the questions just raised can be decided by rather tedious computations [1]. This particular primitive field plays an important role in the solution of the Federbush model [2, 1]. In models that are only partially solvable e.g. for the derivative coupling of a massless, neutral, pseudoscalar particle to a charged spinor field [3] general citeria are needed. We therefore pose the following problem: Let a local Wightman theory in 1 time and n space dimensions be given: $\{\mathscr{H}; U(a, \Lambda); \phi_\alpha(x), W_\mu(x)$ $\alpha = 1, \ldots, l\ \mu = 0, 1, \ldots, n\}$ [4]. Let the linear domain of definition D common to the operators

$$\phi_\alpha(\varphi), \phi_\alpha^*(\varphi) \subset \phi_\alpha(\bar{\varphi})^*, W_\mu(f) \quad \text{where} \quad \varphi, f \in \mathscr{S}(R^{1+n}),$$

invariant under the application of these field operators and $U(a, \Lambda)$ and containing the unique vacuum state Ω be just the set D_1 of quasilocal states (i.e. those states which can be obtained by smearing monomials

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[1] In one time and one space dimension these are exactly the local models with conserved vector or pseudovector currents.