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Representations of the CCR in the $(\phi^4)_3$ Model: Independence of Space Cutoff

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Abstract. The algebra of observables for the renormalized ϕ^4 interaction in 3-dimensional space-time is constructed. It is shown that the von Neumann algebras associated with observables in a bounded region B are independent of the space cutoff which is used in the construction of a Hamiltonian for this interaction. This result is shown to be useful in the construction of a translation invariant ϕ^4 theory in three dimensions. It also gives a physical criterion for the equivalence of non-Fock representations of the canonical commutation relations.

I. Introduction

Recently, there has been some interest in a certain class of non-Fock representations of the canonical commutation relations (CCR) which occurs in a natural way in the construction of a dense domain for a Hamiltonian for the $:\phi^4:$ interaction in $2+1$ dimensional space-time.

This construction was initiated by Glimm [10], who considered an interaction

$$H_\sigma(g) = H_0 + \int :\phi_\sigma^4: (x)\, g(x)\, d^2x + M_\sigma + E_\sigma \tag{1.1}$$

with a momentum cutoff σ (the momenta occurring in the interaction are bounded in absolute value by σ) and with a space cutoff g which is a smooth function with compact support. H_0 is the free Hamiltonian and $\phi_\sigma(x)$ is the cutoff free boson field at time zero:

$$\phi_\sigma(x) = \int_{|k| \leqq \sigma} e^{ikx} (k^2 + m_0^2)^{-1/4} (a^*(k) + a(-k))\, d^2k\,, \qquad m_0 > 0\,.$$

M_σ and E_σ are the mass and the additive counterterms respectively whose definitions are suggested by perturbation theory. In order to define a Hamiltonian $H_\infty(g)$ in the limit $\sigma \to \infty$, Glimm used a modified, truncated version of the formal wave operator. This operator $T_\sigma(g)$ is called a dressing transformation. His construction is summarized in the following

Theorem 1.1 (Glimm [10]). *Let* $\Lambda_\sigma(g) = \left\| H_0^{-1} \int :\phi_\sigma^4: (x) g(x) d^2x\, \Omega \right\|^2$, *where* Ω *is the Fock vacuum. There exists a family* $T_{\varrho\sigma}(g)$ *of dressing*