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Exact Solution of the Schrödinger Equation with a Central Potential

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Abstract. The exact solution of the Schrödinger equation is derived for the case of a central potential under rather weak restriction on it. The solution is given in a form of a simple series which converges strongly and it is suitable for calculation of phase shifts and eigenvalues. Also, as the derivation of the solution is purely algebraic its analytical continuation in the energy or angular momentum complex plane is straightforward.

Introduction

In Ref. [1] an "integro-iteration" method was used to obtain the solution of any second order linear differential equation. This method when applied to the Schrödinger equation gives the solution under rather weak restrictions on the potential $V(r)$.

The procedure starts with the usual transformation of the differential equation into an integral one. Then the function is split in two components, thus obtaining two coupled integral equations. At this point an integration step is inserted and then a two component iteration is performed. The solution is obtained in terms of a *simple* series which is strongly convergent for a large class of potentials.

Although the modification of the procedure for solving the integral equation by a simple iteration looks to be slight, its effect on the result is quite remarkable. To be more explicit: the simple iteration method leads to the Born's expansion. However, as, for $l > 0$, the kernel has two components, the iteration gives multiple series, i.e. the n^{th} term of it has 2^n components. Besides one has to face difficulties with its convergence. On the other hand the Fredholm method is too powerful to be used in the case of a simple second order linear differential equation and bears comparatively too much complexity. For example one has to compute two series in each of which the n^{th} term has $n!$ components. Moreover it seems to demand stronger limitations on the potential than the present method, i.e. as one can see in [2] the minimum conditions on $V(r)$ using