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Tensor Formulation of Spin-1 and Spin-2 Fields

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Abstract. Spin projection operators which constitute a resolution of the identity in the space of second rank tensor wave functions are constructed. These projectors are then used to establish Lagrangian quantum field theories for free massive particles with spin-1 (two equivalent formulations) and spin-2.

1. Introduction

Recently, Aurilia and Umezawa [1, 2] have introduced an elegant systematic and flexible method for constructing quantized field theories for free particles with higher spins. For such particles the method yields a single wave equation, which implies all the usual subsidiary conditions, including symmetry or antisymmetry properties, so that the wave function in question has exactly the number of components appropriate to its spin value. Further, in virtue of these subsidiary conditions, the original equation then reduced to the Klein-Gordan equation for integral spins and to the Dirac equation for half-integral spins. The method of Aurilia and Umezawa represents a decisive advance in the theory of higher spin particles initiated by Dirac [3], Fierz [4], and Pauli [5], developed further by Rarita and Schwinger [6] and many others, and, in fact, excellently reviewed by these authors themselves [2].

The quantization of the various wave equations in the A.U. approach, i.e. the derivation of commutation relations and Feynman propagators is achieved by introducing the operator [7, 8, 9] $d(p)$, known as the Klein-Gordan divisor, which has the property

$$d(p)\,\Lambda(p) = \Lambda(p)\,d(p) = -(p^2 - m^2)\,, \tag{1.1}$$

where the kernel $\Lambda(p)$ is given by the wave equation

$$\Lambda(p)\,\psi = 0\,. \tag{1.2}$$

The operator $d(p)$ is closely related to the spin projection operators $\Theta(s)$ introduced by Fronsdal [10], which project the spin s components out of an arbitrary wave function. In fact $d(p)$ is equal to the projection operator when $p^2 - m^2 = 0$.