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Upper Bounds for Ising Model Correlation Functions

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Abstract. A Griffiths correlation inequality for Ising ferromagnets is refined and is used to obtain improved upper bounds for critical temperatures. It is shown that, for non-negative external fields, the mean field magnetization is an upper bound for the magnetization of Ising ferromagnets.

1. Introduction

For each nonempty subset R of an index set Λ define

$$\sigma_R = \prod_{i \in \Lambda} \sigma_i \tag{1.1}$$

where $\sigma_i = \pm 1, i \in \Lambda$, is a set of Ising spins. In a given configuration of spins $\{\sigma\} = \{\sigma_i : i \in \Lambda\}$, the interaction energy is defined by

$$E\{\sigma\} = -\sum_{R \subset \Lambda} J(R)\,\sigma_R\,. \tag{1.2}$$

Thermodynamic averages of functions $f = f\{\sigma\}$ are defined by

$$\langle f \rangle = \sum_{\{\sigma\}} f\{\sigma\} \exp(-\beta E\{\sigma\}) / \sum_{\{\sigma\}} \exp(-\beta E\{\sigma\}) \tag{1.3}$$

where sums extend over all configurations of spins. We denote

$$\sigma_R \sigma_S = \sigma_{RS} \tag{1.4}$$

where from the Definition (1.1) RS is the set-theoretic symmetric difference $R \cup S - R \cap S$.

For ferromagnetic pair interactions, i.e., $J(R)$ non-negative and zero unless R is a one or a two element subset of Λ (one element subsets corresponding to interactions with an external field), Griffiths [1, 2, 3] proved a number a correlation function inequalities which were subsequently generalized by Kelley and Sherman [4]. For the inter-

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