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Applications of Hyperdifferential Operators to Quantum Mechanics*

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Abstract. In this paper, various applications of the theory of hyperdifferential operators to quantum mechanics are discussed. A concise summary of the relevant aspects of the theory is presented, and then used to derive a variety of operator identities, expansions, and solutions to differential equations.

§ 1. Introduction

The purpose of this paper is to point out various applications of the theory of hyperdifferential operators. In particular we will illustrate how this theory can be used to derive a wide variety of operator identities, expansions, and solutions to differential equations of interest in quantum mechanics.

Hyperdifferential operators are differential operators of infinite order with variable coefficients

$$A = \sum_{m,n=0}^{\infty} a_{mn} z^m \left(\frac{d}{dz}\right)^n . \tag{1.1}$$

The feature of these operators which is of particular relevance in applications is the possibility of defining the symbol of A,

$$\sigma A(z, \xi) = \sum_{m,n=0}^{\infty} a_{mn} z^m \xi^n \tag{1.2}$$

in terms of which we can develop a computational calculus, called the symbolic calculus.

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