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A Remark on the Localisation of a Charged Bose Field

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Abstract. We give two natural definitions for the local field algebras associated with a massive relativistic charged Bose field. These are both covariant, but are relatively antilocal in the sense of Segal and Goodman [1].

They have the same representation of the Poincaré group, but are not in the same Borchers class [2].

1. Introduction

The purpose of this note is to point out that there are two relativistically covariant descriptions of the charged field in terms of local field algebras. These are not essentially the same, as one might expect; rather they are antilocal with respect to each other [1].

We begin by giving the notation.

The Fock space for the charged field is given by $\mathscr{F} = \mathscr{F}_+ \otimes \mathscr{F}_-$ where $\mathscr{F}_\pm$ is the usual symmetric Fock space over $L^2(\mathbb{R}^3, d\Omega)$; $d\Omega = d^3k/2\sqrt{(\boldsymbol{k}^2+m^2)}$. The charged field $\phi(x)$ and its time-derivative, $\pi(x)$, are defined as operator-valued distributions by

$$\phi(x) = (2\pi)^{-3/2} \int_{k^0=\omega(\boldsymbol{k})} [e^{i(k,x)} a_+^*(\boldsymbol{k}) \otimes \mathbf{1} + \mathbf{1} \otimes a_-(\boldsymbol{k})\, e^{-i(k,x)}] \frac{d^3k}{\sqrt{2\omega(\boldsymbol{k})}}$$

$$\pi(x) = i(2\pi)^{-3/2} \int_{k^0=\omega(\boldsymbol{k})} [e^{i(k,x)} a_+^*(\boldsymbol{k}) \otimes \mathbf{1} - \mathbf{1} \otimes a_-(\boldsymbol{k}) e^{-i(k,x)}] \sqrt{\frac{\omega(\boldsymbol{k})}{2}}\, d^3k$$

where $(k,x) = k^0x^0 - \boldsymbol{k}\cdot\boldsymbol{x}$, $\omega(\boldsymbol{k}) = \sqrt{\boldsymbol{k}^2+m^2}$ and $a_\pm^\#(\boldsymbol{k})$ are the annihilation and creation forms on $\mathscr{F}_\pm$:

$$a_\pm(\boldsymbol{k}) : \Psi_\pm(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) \mapsto \sqrt{\frac{n}{2\omega(\boldsymbol{k})}}\, \Psi_\pm(\boldsymbol{k}, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_n)$$

$$a_\pm^*(\boldsymbol{k}) : \Psi(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) \mapsto \sqrt{\frac{2\omega(\boldsymbol{k})}{n+1}} \sum_{i=1}^{n+1} \delta(\boldsymbol{k}-\boldsymbol{k}_i)\, \Psi_\pm(\boldsymbol{k}_1, \ldots, \hat{\boldsymbol{k}}_i, \ldots, \boldsymbol{k}_{n+1})$$

where $\Psi_\pm$ is an n-particle vector in $\mathscr{F}_\pm$.

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