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Free Energy of Gravitating Fermions

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Abstract. We calculate rigorously, in a suitable thermodynamic limit, the free energy of a system of nonrelativistic fermions which interact with attractive r^{-1}-potentials. It is shown that the effective field approximation becomes exact in this limit and results in the temperature-dependent Thomas-Fermi equations.

1. Introduction

The quantum mechanical Hamiltonian

$$H = \sum_{i=1}^{N} \frac{\boldsymbol{P}_i^2}{2M_i} + \sum_{1 \leqq i < j \leqq N} \frac{e_i e_j - \kappa M_i M_j}{|\boldsymbol{x}_i - \boldsymbol{x}_j|} \tag{1.1}$$

describing N particles interacting with r^{-1} potentials is the relevant quantity if weak and nuclear interactions as well as relativistic effects can be neglected. In spite of the vast domain of applicability only few results have been rigorously derived from it, if $N > 2$. Dyson and Lenard [1] have shown that, for $\kappa = 0$, $\sum_i e_i = 0$ and certain combinations of statistics, the ground state energy of (1.1) for large N is proportional to N. Lebowitz and Lieb [2] announced a proof that the free energy F_N then is well-behaved.

Lévy-Leblond [3] proved that, for $\kappa > 0$ and $\sum_i e_i = 0$, the ground state energy for identical fermions is proportional to $N^{7/3}$ for large N.

We propose to calculate exactly the limit $N \to \infty$ of $N^{-7/3} F_N$ for nonrelativistic identical fermions interacting with their gravitational forces. The reason why this can be done is that, owing to the long range of the force, the temperature-dependent Thomas-Fermi equations become exact.

The system exhibits an interesting thermic behaviour which resembles certain features of stars and which has been discussed previously for simplified models [4].

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