Commun. math. Phys. 24, 1—21 (1971)

Differential Vertex Operations in Lagrangian Field Theory*

J. H. LOWENSTEIN

Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania USA

Received July 5, 1971

Abstract. A general framework is derived for studying differential operations in renormalized perturbation theory. The method makes possible a simple, unified derivation of the renormalization group and Callan-Symanzik equations, as well as a direct test for broken symmetries (including broken scale invariance), without the necessity of defining currents and deriving their generalized Ward identites. A second-order differential equation of the Callan-Symanzik type is derived using similar methods.

I. Introduction

Various differential operations on the Green's functions of Lagrangian field theory have proven to be useful tools in investigating the renormalization properties [1, 2] and short-distance behavior [3, 4] of these functions. The aim of the present article is to formulate a simple general framework for studying differential operations (e.g. derivatives with respect to masses and coupling constants) within the context of *BPHZ* [1, 5] renormalized perturbation theory (see [5] for further references). The method of differential vertex operations to be developed in Sections II through IV will make possible (a) a simple and unified derivation of the renormalization group [6, 1] and generalized Callan-Symanzik [3, 4] equations (Sections III B, C) (b) a method for verifying directly whether a given theory possesses broken symmetry (i.e. its truncated Green's functions are symmetric asymptotically for small distances), without introducing currents and generalized Ward identities (Section III D) and (c) the generalization of the Callan-Symanzik equations to second order, thus providing the basis a more detailed description of the asymptotic short-distance behavior of vertex functions (Section IV).

II. Basic Concepts

A. Definition of Differential Vertex Operations

Given a theory with basic fields $A^{(i)}(x)$ and effective Lagrangian

$$\mathscr{L}_{\mathrm{EFF}} = \mathscr{L}_0 + \mathscr{L}_I$$

* Supported in part by the U. S. Atomic Energy Commission under Contract No. AT-30-1-3829.