

Local Observables and Particle Statistics I

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Abstract. We consider the family of those states which become asymptotically indistinguishable from the vacuum for observations in far away regions of space. The pure states of this family may be subdivided into superselection sectors labelled by generalized charge quantum numbers. The principle of locality implies that within this family one may define a natural product composition (leading for instance from single particle states to n -particle states). Intrinsically associated with the n -fold product of states of one sector there is a unitary representation of $P^{(n)}$, the permutation group of n elements, analogous in its role to that arising in wave mechanics from the permutations of the arguments of an n -particle wave function. We show that each sector possesses a "statistics parameter" λ which determines the nature of the representation of $P^{(n)}$ for all n and whose possible values are $0, \pm d^{-1}$ (d a positive integer). A sector with $\lambda \neq 0$ has a unique charge conjugate ("antiparticle" states); if $\lambda = d^{-1}$ the states of the sector obey para-Bose statistics of order d , if $\lambda = -d^{-1}$ they obey para-Fermi statistics of order d . Some conditions which restrict λ to ± 1 (ordinary Bose or Fermi statistics) are given.

I. Introduction

We continue here our discussion of the superselection structure of elementary particle physics. The setting has been described in some detail in [1] and [2] so a few remarks may suffice here. We consider the net \mathfrak{A} of algebras of local observables¹ as the basic mathematical object in the theory, and we consider a set \mathcal{S} of states over \mathfrak{A} as representing, in an appropriate idealization, the states of interest in elementary particle physics.

¹ This is a correspondence $\mathcal{O} \rightarrow \mathfrak{A}(\mathcal{O})$ between finite regions in space-time and C^* -algebras. As in [2] we shall always take \mathcal{O} to be a closed double cone (the intersection of a closed forward light cone with a closed backward light cone). The symbol \mathcal{O}' denotes the causal complement of \mathcal{O} i.e. the infinitely extended region containing all points spacelike to \mathcal{O} . The algebra $\mathfrak{A}(\mathcal{O}')$ is defined as the C^* -algebra generated by all $\mathfrak{A}(\mathcal{O}_i)$ with \mathcal{O}_i any double cone spacelike to \mathcal{O} . The C^* -algebra generated by all $\mathfrak{A}(\mathcal{O})$ is again denoted by \mathfrak{A} .