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Gentle Perturbations

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Abstract. By introducing a specific type of perturbation, A, in the Hamiltonian, we define a class of gently perturbed states, $\varrho_{\beta,A}$, of a canonical ensemble, ϱ_β. The perturbations are chosen so as to preserve a relationship of the form $\varrho_{\beta,A} \leqq \text{constant} \times \varrho_\beta$. Applications in ergodic theory and phase transitions are described.

1. Introduction

It is not difficult to give examples wherein the state of a dynamical system is radically altered by the introduction of a perturbation. It is our purpose however, to investigate the effects on a canonical ensemble of a specific class of very weak perturbations, the choice being made so as to preserve a certain relationship with the unperturbed state. The relationship is known to be of use in studying problems of ergodic theory and of phase transitions, and we explicitly mention important, "non-sovable" models to which our results apply.

We are primarily concerned with infinite volume, quantum mechanical systems, and the C^*-algebra formalism is used. The system is thus assumed to be describable by the C^*-inductive limit [1] $\mathfrak{A}$ of an increasing sequence of finite volume subsystems $\mathfrak{A}_n$, i.e. sub-C^*-algebras, which are C^*-isomorphic to[1] $B(\mathscr{H}_n)$ for some sequence $\mathscr{H}_n$ of Hilbert spaces. To simplify the notation, we will identify $\mathfrak{A}_n$ with $B(\mathscr{H}_n)$ at will.

If H is a self-adjoint operator on a Hilbert space $\mathscr{H}$, with generalized resolution of the identity $\{E_\lambda \mid -\infty < \lambda < \infty\}$, we shall mean by $e^{\beta H}$ the self-adjoint operator with domain:

$$D(e^{\beta H}) = \left\{ \psi \in \mathscr{H} \,\middle|\, \int_{-\infty}^{\infty} e^{2\beta\lambda} d\|E_\lambda \psi\|^2 < \infty \right\}$$

and definition:

$$e^{\beta H} : \psi \in D(e^{\beta H}) \to \int_{-\infty}^{\infty} e^{\beta\lambda} d(E_\lambda \psi) .$$

The meaning of the integral is that of [§ 29.2; 2].