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On the Harish-Chandra Condition for First-Order Relativistically-Invariant Free Field Equations

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Abstract. It has hitherto been accepted that the degree of the Harish-Chandra condition applying to single-mass equations of arbitrary spin is determined by the maximum spin appearing in the representation of $SL(2, C)$ which acts on the field. The present paper demonstrates a fallacy in the published arguments which lead to the above conclusion, and gives the correct conclusion which can be deduced from the hypotheses. A counter-example of an irreducible, single-mass, spin 3/2 equation which does not satisfy the accepted theory is provided in an appendix.

Relativistically-invariant free-field equations of the form

$$(\beta_\mu \partial^\mu + \mathrm{im})\,\psi = 0 \tag{1}$$

have been studied by many authors [1–5, 8, 11–14] (see in particular Corson [13] and Takahashi [14] for complete bibliography). In the case that (1) yields solutions with only one mass (apart from sign), Harish-Chandra has shown [1] that the β-matrices must satisfy

$$(\beta_\mu p^\mu)^n - p^2(\beta_\mu p^\mu)^{n-2} = 0 \tag{2}$$

for some finite integer n. If one regards (2) as a polynomial in the variables p_0, p_1, p_2, and p_3 [12], the coefficients must vanish, and:

$$\sum_{\substack{\text{permutations}\\ \{1,\ldots,n\}}} (\beta_{\mu_1}\beta_{\mu_2} - g_{\mu_1\mu_2})\,\beta_{\mu_3}\cdots\beta_{\mu_n} = 0\,. \tag{2a}$$

Umezawa and Visconti [3] claimed that (2) is satisfied for a minimum value of n, $n = 2s_0 + 1$, where s_0 is the maximum spin appearing in the representation of $SL(2, C)$ which acts on ψ (we denote this representation by $S(\Lambda)$). This result, which seems to be generally accepted, is based on

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