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Strict Convexity ("Continuity") of the Pressure in Lattice Systems

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Abstract. It is shown that the pressure is a strictly convex function of the translationally invariant interactions (under certain mild restrictions on the long-range part of these interactions) for classical and quantum lattice systems, by demonstrating that two distinct interactions can never lead to the same translationally invariant equilibrium state. This generalizes a previous result that the pressure is a continuous function of density at fixed temperature.

I. Introduction

The pressure P has been shown to be a continuous function of the density ϱ, at constant temperature T, for certain classical [1]–[5] (p. 58), and quantum [6] systems of interacting particles in equilibrium. Since P is a convex increasing function of the chemical potential μ and $\varrho = \partial P/\partial \mu$, it is evident that continuity of P as a function of ϱ is equivalent to the strict convexity of P as a function of μ: the graph of $P(\mu)$ has no linear segments. It is then rather natural to ask whether P is not also strictly convex in T (and thus a continuous function of the entropy) at fixed μ, or in other similar "intensive" thermodynamic variables.

In this paper we shall show that for classical and quantum lattice gases, P is a strictly convex function of any linear parameter in the interaction Hamiltonian or potential energy, provided this interaction possesses translational invariance and satisfies certain other mild restrictions. The argument makes use of the relationship ([5], pp. 184ff.) between the statistical "state" of such a system (the set of probability distributions or reduced density matrices for finite sets of lattice sites) and tangent planes to the pressure regarded as a function of the interaction Φ (further details are given below). The existence of a first-order phase transition is characterized by the possibility of at least two distinct planes tangent to P for a single interaction, which is to say at least two possible states (e.g., "liquid" and "vapor"). By contrast, if P were not a strictly convex function of the interactions, one could find two different interactions corresponding to the same state. We shall show that for the