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# Spherical Functions of the Lorentz Group on the Two Dimensional Complex Sphere of Zero Radius

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**Abstract.** Spherical functions of the Lorentz group with respect to the horyspheric subgroup are derived and their relation to Gelfand's homogeneous functions are discussed.

## Introduction

There exists a number of homogeneous spaces whose group of motion may serve for the definition of the Lorentz group. Of these homogeneous spaces the most familiar is the three dimensional hyperboloid. It has turned out, however, that in certain respects it is expedient to treat the Lorentz group as a group of motion of the two-(complex) dimensional complex sphere $\boldsymbol{S}^2 = S_1{}^2 + S_2{}^2 + S_3{}^2$. Namely, it has been pointed out by H. Joos and R. Schrader [1] and by M. Huszár and J. Smorodinsky [2] that if the Lorentz group is considered in this spirit, matrix elements of its unitary representation take a rather simple form.

A three dimensional complex vector $\boldsymbol{S}$ is the self-dual part of the Lorentz covariant antisymmetric tensor $S_{\mu\nu}$, i.e. $S_k = S_{0k} + \frac{i}{2}\varepsilon_{klm}S_{lm}$, $(k, l, m = 1, 2, 3)$. Since the real and imaginary part of $\boldsymbol{S}$ transform like the electric and magnetic field, respectively, the invariance of $\boldsymbol{S}^2 \sim (\boldsymbol{E} + i\boldsymbol{B})^2$ under proper Lorentz transformations is evident. And conversely, it can be proved [3] that the connected part of three dimensional complex rotation group is isomorphic to the proper Lorentz group.

## 1. Little Groups on the Complex Sphere of Zero and Non-zero Radius

Let us associate to a three dimensional complex vector $\boldsymbol{S} = (S_1, S_2, S_3)$ the matrix $\hat{S} = \begin{pmatrix} S_3 & S_1 - iS_2 \\ S_1 + iS_2 & -S_3 \end{pmatrix}$. Under $g \in SL(2, C)$ $\boldsymbol{S}$ transforms as

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