Commun. math. Phys. 23, 100—116 (1971)

Classical Systems and Observables in Quantum Mechanics

Holger Neumann
Institut für Theoretische Physik (I) der Universität Marburg (Lahn)

Received March 29, 1971

1. Introduction

In [1] Chap. III Ludwig gave the definition of coexistent effects in a quantum mechanical system. Coexistence of effects will be discussed in this paper. Special attention will be called to the connection between the notion "observable" and the notion "classical system". It will be proved in particular that the set of classical effects is coexistent and therefore the convex range of observables consists of coexistent effects.

We shall stick closely to the notation introduced in [1]. The quantum mechanical system is described by subsets of an ordered separable Banach space B and its dual B', which satisfy several axioms. The subset

$$K = \{X/X \in B,\ X \geqq 0,\ \|X\| = 1\}$$

of B represents the ensembles, and the subset

$$\hat{L} = \{Y/Y \in B',\ Y \geqq 0,\ \|Y\| \leqq 1\}$$

of B' represents the effects of the system (yes-no-experiments). The probability to measure the effect $F \in \hat{L}$ in the ensemble $V \in K$ is expressed by the value $\langle V, F\rangle$ of the functional F on B. G denotes the set of extreme points of $\hat{L}$, the set of decision effects. G is a complete, orthocomplemented, weakly modular lattice.

Perhaps it is convenient to keep in mind the representation of B and B' by Hermitean operators in a separable Hilbert space. This representation is valid for irreducible quantum mechanical systems (no superselection rule).

B is the set of Hermitean trace class operators.

B' is the set of bounded Hermitean operators.

($\geqq$ is the usual order of Hermitean operators.)

$K = \{V \in B/V \geqq 0, \mathrm{tr}(V) = 1\}$, $\hat{L} = \{F \in B'/0 \leqq F \leqq 1\}$.

G is the lattice of projection operators.

$\langle V, F\rangle = \mathrm{tr}(V \cdot F)$