Commun. math. Phys. 23, 87—99 (1971)

Free Energy in a Markovian Model of a Lattice Spin System

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Received March 25, 1971

Abstract. A Markov process which may be thought of as a classical lattice spin system is considered. States of the system are probability measures on the configuration space, and we study the evolution of the free energy of these states with time. It is proved that for all initial states the free energy is nonincreasing and that it strictly decreases from any initial state which is shift invariant but not an equilibrium state. Finally we show that the state of the system converges weakly to the set of Gibbsian Distributions for the given interaction, and that all shift invariant equilibrium states are Gibbsian Distributions.

1. Introduction

In this paper we study a model of a classical lattice spin system which is also a Markov process. We let Z be the integers and consider the lattice Z^ν. At each point of the lattice we have a particle which is spinning either up of down. Thus the configuration of spins can be represented by functions, ξ, from Z^ν into $\{-1, 1\}$ with the interpretation that the spin at the site x is up (down) if $\xi(x) = 1(-1)$. For each subset R of Z^ν we have a number J_R, and throughout this paper it will be assumed that

$$J_R = J_{R+x} \quad \text{for all} \quad x \in Z^\nu, \tag{1.1a}$$

and

$$\begin{array}{l}\text{there is a positive integer } L \text{ such that if } 0 \in R \\ \text{and } R \text{ is not contained in } [-L, L]^\nu \text{ then } J_R = 0\,.\end{array} \tag{1.1b}$$

Expressions like $[-L, L]^\nu$ will always mean $[-L, L]^\nu$ restricted to Z^ν.

Let $\sigma_R(\xi) = \prod_{x \in R} \xi(x)$ and let $\beta > 0$. Then we define

$$c(x, \xi) = \exp\{\beta \sum_{R \ni x} J_R \sigma_R(\xi)\}\,.$$

The Markov process, $\xi(t)$, can then be described intuitively as follows: if at time t the configuration is $\xi(t)$, then the particle at x reverses

[1] This work was done while the author was a postdoctoral fellow in the Adolph C. and Mary Sprague Miller Institute for Basic Research in Science.