Commun. math. Phys. 23, 37—48 (1971)

Wave Operators for Classical Particle Scattering

Barry Simon
Departments of Mathematics and Physics, Princeton University
Princeton, New Jersey, USA

Received June 1, 1971

Abstract. We discuss the fundamentals of classical particle scattering of a two body system in forces which are 0 $(r^{-2-\varepsilon})$ at infinity along with their Lipshitz constants. We prove asymptotic completeness for this two-body case. Of particular interest is the fact that in the absence of control on Lipshitz constants at ∞, two solutions of the interacting equation may be asymptotic to the same free solution at $-\infty$.

§ 1. Introduction

During the past fifteen years, one of the most studied areas of mathematical physics has been the rigorous foundations of scattering theory. There have been studies of the quantum scattering of two bodies in short range [1] and long range forces [2], of n-body quantum systems [3], of potential scattering with free dynamics given by relativistic wave equations [4], of accoustical scattering [5], of scattering in the general theory of quantum field [6], of scattering of quantized particles in the presence of unquantized external electromagnetic fields [7], of scattering in model field theories [8] and of classical field theoretic scattering [9]. Suprisingly there has been very little study of the most classical of scattering problems: the scattering of classical particles moving in Newtonian force fields. Hunziker [10] and Cook [11] have briefly studied classical scattering by taking L^2 of phase space and using the Hilbert space techniques developed for quantum particle scattering [1, 3]. In this note, we wish to study classical particle scattering directly in phase space; this approach is in many ways more natural than Hunziker's construction involving L^2 of phase space. As we will discuss in § 7, up to sets of measure zero, Hunziker's results imply most of ours but the two methods of proof are very different.

The basic existence theorem we will prove is that given a solution of the free equation $\vec{r}_o(t) = \vec{a} + \vec{b}t$, there exists a solution, $\vec{r}(t)$, of the interacting equation so that $\lim_{t\to-\infty} |\vec{r}(t) - \vec{r}_o(t)| + |\dot{\vec{r}}(t) - \dot{\vec{r}}_o(t)| = 0$. We view this as an existence theorem for solutions with boundary conditions at

Research partially supported by U.S. Air Force under contract AF49(638) 1545.