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On the Structure of Analytic Renormalization

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Abstract. A direct proof is given that analytic renormalization has an additive structure and hence may be implemented by counterterms in the Lagrangian.

§ 1. Introduction

Of the various types of renormalization in perturbative quantum field theory we will here accept as basic the *additive* approach of Bogoliubov [1]. In this method formally infinite quantities are subtracted from a divergent Feynman amplitude to produce a finite renormalized result. Physics enters because these subtractions may be implemented in the field theory by inserting counterterms into the interaction Lagrangian; the counterterms are then related to renormalizations of mass, charge, etc.

The particular implementation of the additive aproach given by Bogoliubov has two difficulties: the recursive definition of the renormalized amplitudes makes them difficult to compute, and the proof that the result is finite is very complicated (see [1–3]). *Analytic renormalization* ([4, 5]) removes these difficulties, to a large extent, by giving a simple prescription which obviously yields a finite result. On the other hand, to establish the connection with physics (specifically, to show that analytic renormalization is equivalent to additive renormalization) it was necessary in [4, 5] to use the recursive definition as a starting point. In the present paper we avoid this problem and show directly that analytic renormalization may be implemented by counterterms in the Lagrangian.

Some work related to these goals has previously appeared. In [6] Westwater gives a definition of analytic renormalization which he shows to be implementable by counterterms (and to be equivalent to the definition of [4, 5]). His definition is not recursive, but, like the Bogoliubov prescription, does involve modifying the integral by modification of (i.e., subtractions from) the integrand. The present paper is more in the spirit of [4, 5]; the counterterms, like the renormalized amplitude itself,