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A Class of Analytic Perturbations for One-body Schrödinger Hamiltonians

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Abstract. We study a class of symmetric relatively compact perturbations satisfying analyticity conditions with respect to the dilatation group in R^n. Absence of continuous singular part for the Hamiltonians is proved together with the existence of an absolutely continuous part having spectrum $[0, \infty)$. The point spectrum consists in $\mathbb{R} - \{0\}$ of finite multiplicity isolated energy bound-states standing in a bounded domain. Bound-state wave functions are analytic with respect to the dilatation group. Some properties of resonance poles are investigated.

Introduction

The recent developments of scattering theory for long range potentials ([1–3]) have made urgent to find new techniques for the spectral analysis of Schrödinger Hamiltonians. By new we mean independent of a short range hypothesis implying e.g. existence of the usual wave operators or (almost equivalently) solvability of the Lippman-Schwinger equation. Many results have been found recently by using methods related to Putnam's positive commutator theorems (to our knowledge the most recent of them can be found in [4, 5] which also contains many references). We present here a different method based upon rather weak analyticity conditions on the potentials. These conditions allow analytic continuation on the unphysical sheet for sufficiently many expectation values of the resolvent; from this all qualitative results on the spectrum can be deduced. Our conditions allow velocity dependent perturbations such as spin-orbit couplings or electromagnetic fields. As a consequence we don't expect absence of positive discrete spectrum. A part of this work will be concerned with a study of positive energy bound-states and corresponding eigenfunctions. Finally, we investigate properties of poles on the unphysical sheet.

I. Dilatation Analytic Potentials

The dilatation group in $L^2(R^n)$ is defined as

$$(U(\theta)\Phi)(x) = \iota^{n\theta/2}\Phi(e^\theta x) \qquad \theta \in R,\ \Phi \in L^2(R^n). \tag{1}$$