Commun. math. Phys. 22, 223—237 (1971)

Some Connections between Ground States and Temperature States of Thermodynamical Systems

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Received February 16, 1970

Abstract. We investigate the properties of ground states of thermodynamical systems as limits of temperature states; we enlarge the algebra of observables in order to define a $*$-automorphism relevant to the study of K.M.S.-states and derive an usefull theorem of transitivity of $*$-automorphisms within the locally normal pure states of a quasilocal algebra.

§ 1. Introduction

In contrast to the situation encountered in statistical mechanics, states of field theory are generally pure states. This fact is usually explained by saying that states of field theory correspond to zero temperature and zero density.

It is our goal in this note to make this statement a little more precise and to study the behaviour of temperature states when the temperature goes to zero.

If we assume that the limits of temperature states exist in the appropriate sense, then one can derive some properties of these limits and recognize the well-known properties of a ground state; roughly speaking type I von Neumann algebras with positive hamiltonian.

We shall describe the behaviour of the states with respect to the temperature by a family of $*$-automorphisms; of course, these cannot be automorphisms of the algebra itself but merely of one of its natural enlargements. This can be achieved by extending the K.M.S. states we consider to pure states of the enlarged algebra, and by proving a theorem of transitivity of $*$-automorphisms within the locally normal pure states of a quasi-local algebra.

In general, we only have an existence theorem for these $*$-automorphisms and it is not sufficient to give precise results about the existence of K.M.S. states. Nevertheless, we give an example where we can construct this automorphism explicitly.

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