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Rotation does not Enhance Mixing in the Mixmaster Universe*

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Abstract. We investigate closed rotating cosmologies to determine if rotation leads to an enhancement of causal mixing proposed by Misner to guarantee the homogeneity of such models. We conclude that rotation cannot lead to significantly more efficient mixing than occurs in non-rotating models. Since arguments presented by Doroshkevich and Novikov and calculations made by Chitre give very small probability of mixing in non-rotating models, we therefore conclude that a plausible explanation of the homogeneity of the universe cannot be found within the framework of classical General Relativity. Such an explanation may lie in quantum effects on mixing near the singularity.

I. Introduction

We consider here the phenomenon of "causal mixing", first postulated by Misner (1969a) for the Mixmaster universe (a closed non-rotating model). At any fixed time or volume epoch Ω near the singularity, the Mixmaster universe can be specified by initial conditions β_+, β_- (shape anisotropy) and $\not{p}_+$, $\not{p}_-$ (expansion rate anisotropy). Chitre (1970) has shown that for certain subsets of initial conditions some null-geodesics will circumnavigate the corresponding universe and that a probability for this can be computed since the Einstein equations lead to a natural measure on initial conditions $(\beta_\pm, \not{p}_\pm)$. Doroshkevich and Novikov (1970) and Chitre (1971) find that the probability of removing horizons in any one direction is very low. That is, the possibility exists, but the probability of such behavior is quite small.

In this paper we discuss models with rotation. The basic idea is that in such models the rotation may move the mixing direction so that effective mixing in many directions is possible. We shall show that rotation leads to slightly more efficient mixing, but the improvement is in no

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