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Analyticity of the Partition Function for Finite Quantum Systems

H. D. MAISON*
CERN-Geneva

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Abstract. The partition function $Z(\beta, \lambda) = \operatorname{Tr} e^{-\beta(T+\lambda V)}$ for a finite quantized system is investigated. If the interaction V is a relatively bounded operator with respect to the kinetic energy T with T-bound $b<1$, $Z(\beta, \lambda)$ is shown to be a holomorphic function of β and λ for

$$|\arg \beta| < \operatorname{arc\,tg} \frac{\sqrt{1-b^2|\lambda|^2}}{b|\lambda|} \quad \text{and} \quad |\lambda| < b^{-1}.$$

For $b=0$ $Z(\beta, \lambda)$ is an entire function of λ and holomorphic in β for $\operatorname{Re} \beta > 0$.

1. Introduction

The partition function for a canonical ensemble is defined to be $\operatorname{Tr} e^{-\beta H}$, where H is the Hamiltonian of the system under consideration. We are dealing in this work with finite systems only (i.e., a finite number of particles in a box of finite volume), for which H can be decomposed into the kinetic energy T and the interaction energy λV (λ = coupling constant). In the Schrödinger representation T is given by the $3n$-dimensional Laplace operator $T = -\Delta_{3n}$ (n = number of particles) with suitable boundary conditions to make it self-adjoint; V is usually represented by a set $\{V_m(\underline{x}_1, \ldots, \underline{x}_m)\}$ of m-body potentials. In the definition of the partition function $Z(\beta, \lambda) = \operatorname{Tr} e^{-\beta(T+\lambda V)}$ we encounter immediately two mathematical problems:

i) find conditions on V under which it is possible to define a semi-bounded self-adjoint Hamiltonian $H = T + \lambda V$;

ii) show that $\operatorname{Tr} e^{-\beta H}$ exists for $\beta > 0$.

If this has been achieved we may further ask, what are the analytical properties of $Z(\beta, \lambda)$:

a) is $Z(\beta, \lambda)$ an analytic function of λ at $\lambda = 0$, what is the radius of convergence for the perturbation series?

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* On leave of absence from the Max-Planck-Institut für Physik und Astrophysik, München.