Commun. math. Phys. 22, 162—165 (1971)

On Stable Potentials

N. ANGELESCU, G. NENCIU, and V. PROTOPOPESCU
Institute for Atomic Physics, Bucharest, Romania

Received April 23, 1971

Abstract. Examples are given of one- and two-dimensional stable potentials which cannot be decomposed into the sum of a non-negative function and a continuous stable potential.

§ 1

In his book on Statistical Mechanics [1], D. Ruelle raised the question whether every measurable stable potential on R^ν can be decomposed into the sum of a continuous function of positive type and a non-negative function. In a recent paper [2], Lenard and Sherman studied a class of step potentials on R^1 and found inside this class an example of stable potential which cannot be decomposed in this manner. Moreover, they were able to change this potential into a continuous stable potential preserving however the indecomposability property.

This note is concerned with finding further examples of indecomposable stable potentials. The idea is that for a subclass of the step potentials considered in [2], even a weaker decomposability requirement cannot be satisfied. Namely, we are looking for potentials which cannot be written as the sum of a continuous stable potential and a non-negative function. This enables us to considerably simplify the indecomposability proof and, moreover, to find a two-dimensional example. Of course, our examples will consist of surely non-continuous potentials.

§ 2

Let us consider the two-parameter family of potentials $\varphi_{t,d}: R^\nu \to R$, $0 \leqq t \leqq 2$, $d \geqq 0$, defined through:

$$\varphi_{t,d}(x) = \begin{cases} d & \text{for} \quad 0 \leqq |x| \leqq t \\ -1 & \text{for} \quad t < |x| < 2 \\ 0 & \text{for} \quad 2 \leqq |x| \end{cases} \tag{1}$$

$$\varphi_{2,d}(x) = \begin{cases} d & \text{for} \quad 0 \leqq |x| \leqq 2 \\ 0 & \text{for} \quad 2 < |x| \end{cases} \tag{1'}$$