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Ergodic Properties of a Semi-infinite Hard Rods System

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Abstract. A semi-infinite hard rods system in thermodynamic equilibrium is proved to be a K-system.

§ 1. Introduction

The ergodic hypothesis asserts that the asymptotic time average of any summable function is identical to its integral with respect to the probability measure governing the statistics of the system. It is easy to see that this last condition is equivalent to that all invariants under the evolution operator are constant "almost everywhere".

For finite classical systems the number of particles and the total energy are invariants under the evolution operator so that the canonical and grand-canonical equilibriums cannot be ergodic states. However the microcanonical equilibrium is expected to be ergodic for a large class of interaction potentials. In fact the ergodicity of the "Boltzmann-Gibbs" model (hard spheres model) has been recently proved by Sinaï [1].

Unfortunately the ergodicity of systems with a finite number of degrees of freedom is not of great interest in classical statistical mechanics. Indeed the thermodynamic limit gives rise to configurations with an infinite number of particles. In compensation one may conjecture that the thermodynamic equilibrium of infinite classical systems is ergodic (at least for a large class of potentials) for the following reason: the particle number and the total energy are not defined, so that one may think that constants are the only invariants. Precisely we propose in this paper to prove the ergodicity of a somewhat simple model in the thermodynamic limit.

A simple manner to describe a state of a system in the thermodynamic limit is to consider a probability measure on a space of so called "locally finite" configurations [2]. In this context the equilibrium state is unambiguously defined at low activity z for a large class of interaction potentials [3]. Moreover in the case of hard core one-dimensional