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Quantum Stochastic Processes III

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Abstract. We construct an example of a quantum stochastic process with a non-zero, linear, time-independent source, for a massive scalar Boson field in four space-time dimensions. Also we study in detail a similar process with only a single degree of freedom.

§ 1. Introduction and Summary of Results

In an earlier paper [1] we laid the foundations for a theory of quantum stochastic processes, in order to provide a mathematical framework for the description of the evolution of a free photon field in the presence of localised external sources and localised absorptive photon detectors. At the end of that paper a model was constructed which was adequate for arbitrary Boson fields provided sources were absent.

In this paper we construct an example with a non-zero, linear, time-independent source. This is done for a massive scalar Boson field in four space-time dimensions, but formally we are describing the same situation as has been considered non-rigorously in quantum optics. The theory has two space cut-offs, corresponding to the finite extents of the particle source (or sources) and the particle detector (or detectors). As is conventional in quantum optics we suppose that the particle detector is purely absorptive, so the theory is not strictly local.

The same construction allows us to consider the behaviour of a displaced harmonic oscillator with external damping. By studying the time evolution of the coherent states we prove that all initial states converge to a unique equilibrium state, which is a pure coherent state and a slight perturbation of the natural ground state of the oscillator.

As far as the technical details are concerned we point out that for time-independent linear sources the complications can be reduced by making a canonical transformation of the Hilbert space. However, since a realistic model must incorporate time-dependent sources, that method would lead to the necessity of studying randomly fluctuating canonical transformations. With the method we adopt, however, the inclusion of fluctuations in the source is a relatively straightforward matter.

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