Commun. math. Phys. 21, 284—290 (1971)

A Multiplicity Theorem for Representations of Inhomogeneous Compact Groups

H. D. DOEBNER and J. TOLAR*

International Centre for Theoretical Physics, Trieste, Italy, and the University of Marburg, Fed. Rep. Germany

Received August 14, 1970

Abstract. The problem of finiteness of multiplicities of irreducible unitary representations of a compact subgroup is considered for decompositions of irreducible unitary representations of locally compact groups. A simple solution is found for inhomogeneous compact groups and for a physically interesting class of groups with a non-abelian radical.

I. Introduction

We want to discuss the following problem: let $U(G)$ be any irreducible unitary representation of a Lie group G with K being a maximal compact subgroup of G. Decompose $U(G)$ with respect to K, i.e., $U(G) \downarrow K$, and ask: for which groups G are the multiplicities of all irreducible unitary representations $U^{[\alpha]}(K)$ in the decomposition of $U(G)$ finite for all α, where α labels the irreducible unitary representations of K. Non-compact Lie groups G possessing this property are sometimes called "groups which admit a large compact subgroup" (Ref. [1], p. 641).

The decomposition $U(G) \downarrow K$ is needed for physical applications of dynamical groups, which are in general non-compact embeddings G of a compact semi-simple symmetry group K' possessing an irreducible unitary representation $U(G)$ such that $U(G) \downarrow K' = U_{\mathrm{red}}(K')$, where $U_{\mathrm{red}}(K')$ is a given reducible unitary representation of K'. The simplest embeddings of K' are those in which K' is isomorphic to the maximal compact subgroup K of G.

The simply connected embeddings G can be classified using the Levy-Malcev decomposition $G = N \circledS S$, where N and S are simply connected Lie groups, the Levy factor S being semi-simple and the radical N solvable. Because K' is semi-simple and compact, it has to be embedded in S, $K' \subset S$, and we shall distinguish the following cases (T_n is an n-dimensional abelian group):

* On leave of absence from Faculty of Nuclear Science, Czech Technical University, Prague, Czechoslovakia.