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Unitary Renormalization of $[\varphi^4]_{2+1}$ ★

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Abstract. The two-space dimensional φ^4 interaction is renormalized by unitary transformation. A sequence of unitary operators is defined which transform a sequence of cut-off Hamiltonians, arranged in order of increasing cut-off energy, to a sequence of operators converging strongly on a dense set of states. The proof is outlined, calculations leading to required L^2 estimates on the kernels of a finite number of diagrams are not here detailed.

In this paper the two-space dimensional φ^4 theory is treated by the methods initiated in references [1] and [2]. This model has been renormalized by Glimm [3, 4] and essentially the present treatment, using unitary transformation, is a "minimal modification" of the method of Glimm giving rise to a unitary transformation. Arranging a sequence of cut-off Hamiltonians in order of increasing cut-off energy, we will construct a sequence of unitary operators, such that the transformed Hamiltonians converge strongly on a dense set of states. It is expected that the sequence of unitary operators do not converge, and give rise to the same representation of the field operators as the transformation of Glimm [5]. We also collect some estimates for the norm of polynomials in the field operators restricted to states with given numbers of particles; particularly obtaining expressions that are useful when the polynomials are momentum conserving. It is expected that these estimates will be useful in future work; here they replace estimates on the kernels of arbitrarily complicated graphs in Glimm's procedure.

We start by presenting estimates on the norms of certain operator expressions. Let

$$O = \sum_{k_i, p_j} K(k_1, \ldots, p_s)\, a_{k_1}^* \ldots a_{k_R}^* \ldots a_{p_1} \ldots a_{p_S} \tag{1}$$

be an operator constructed from the annihilation and creation operators of boson and/or fermion fields. (The number of fermion operators among the creation and annihilation operators is assumed the same in each term.) There is a sum over momenta as the fields are constructed in a

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