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Gauge Invariance and the Generalized Bondi-Metzner Algebra

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Abstract. We examine the algebraic meaning of the Electromagnetic gauge invariance and show that it leads to the new concepts of gauged operators, gauged representations and hence to infinite dimensional extensions of Lie algebras. In particular we prove that the generalized Bondi-Metzner algebra can be interpreted as a gauged Lorentz algebra related to the Electromagnetic gauge.

I. Introduction

It is well known that Quantum Mechanical states are phase invariant. The reason for this fact is that in Quantum Mechanics the state functions are not directly measurable quantities but only the absolute value of the scalar product, $|(f, g)|$ which is invariant under the replacements

$$f_1 = e^{i\theta} f, \quad g_1 = e^{i\psi} g \qquad (\theta, \psi \text{ reals}).$$

As a result of this invariance we have to consider in Quantum Mechanics rays of functions and therefore also operator rays and ray representations of Lie algebras [1, 2].

However in physics we deal with another important case in which we use quantities which are not directly measurable. This case is the vector potential formalism of the Electromagnetic field. As in the quantum case this formalism is not determined uniquely by the measurable quantities and this enables us to gauge the vector potentials by quantities of the form

$$\left(\operatorname{grad} f, \frac{\partial f}{\partial t}\right)$$

(f well behaved function).

In this paper we show that the analogy between the phase invariance of the quantum wave functions and the gauge invariance of the Electromagnetic potentials can be pushed further and lead us to consider naturally, the new concepts of gauged operators, gauged representations and hence infinite dimensional extensions of Lie algebras.