Commun. math. Phys. 21, 171 – 191 (1971)

Quasi-Equivalence of Quasi-Free States on the Weyl Algebra

A. VAN DAELE*
Institute of Mathematics, University of Oslo, Oslo

Received November 5, 1970

Abstract. A necessary and sufficient condition for quasi-equivalence of quasi-free factor states over the Weyl algebra is proved. The essential part of this paper is closely related to the work of Powers and Størmer ôn the Clifford algebra.

1. Introduction

In this paper we study the quasi-equivalence of quasi-free states of the canonical commutation relations. It is well known that all irreducible representations of these relations for finite systems are unitarily equivalent [1, 2] and that this theorem fails in the case of an infinite system. The algebraic approach to this problem was first given by Kastler [2]. We follow the same method and study the problem of equivalence in terms of states on the C^*-algebra (Weyl algebra) associated with the canonical commutation relations.

In particular we use the C^*-algebra $\overline{\Delta(H, \sigma)}$, built on a symplectic space (H, σ), as introduced by Manuceau [3].

Quasi-free states of the canonical commutation relations were introduced by Robinson [4]. These states were intensively studied by Manuceau and Verbeure [5] who introduced their C^*-algebraic formulation. In this work we study the quasi-equivalence of such quasi-free states. Our approach is very closely related to the work of Powers and Størmer [6] on quasi-equivalence of gauge invariant quasi-free states of the canonical anticommutation relations. Together with Verbeure we proved a necessary and sufficient condition for two pure quasi-free states on the Weyl algebra to be unitarily equivalent [7]. To find a criterium in the case of more general quasi-free states, we used the idea of Powers and Størmer and reduced the latter problem to the case of pure states.

* Aspirant van het Belgisch N. F. W. O. On leave from the University of Louvain (Belgium).