Commun. math. Phys. 21, 150—163 (1971)

Linear Kinematical Groups

VITTORIO GORINI*

Institut für Theoretische Physik (I) der Universität Marburg, Germany

Received November 12, 1970

Abstract. We prove a theorem which states that in an $(n+1)$-dimensional space-time ($n \geqq 3$) the only linear kinematical groups which are compatible with the isotropy of space are the Lorentz and Galilei groups. The special cases $n=1$ and $n=2$ are also briefly discussed.

1. Introduction

We prove in this paper that in an $(n+1)$-dimensional space-time ($n \geqq 3$) the only non trivial linear kinematical groups which are compatible with the isotropy of space are the Lorentz and Galilei groups.

Related to ours are the papers by Lalan [1] and by Bacry and Lévy-Leblond [2]. Lalan's conditions are however much more restrictive than ours in that he assumes at the outset a Lie group structure and he requires the set of special velocity transformations to be invariant under space rotations. As to the approach of Ref. [2], it is more general than ours because space-time translations are considered as well and no restriction to linearity is introduced. On the other hand, the Lie group assumption is still used and, furthermore, invariance under parity and time reversal is required.

In Section 2 we collect some notations. In Section 3 we discuss our assumptions. In Section 4 we state and prove our theorem and mention its extension to the inclusion of space reflection, as a corollary[1]. In Section 5 the cases $n=1$ and $n=2$ are discussed. For $n=1$, some specific counterexamples are listed, which prove that the result no longer holds. As for the case $n=2$, it is seen to hold if space reflection is allowed for, or under restriction to connected Lie groups.

2. Notations

Let m and n be positive integers. We use the standard notations $GL(n+1, R)$ for the group of all $(n+1) \times (n+1)$ real non singular matrices

* On leave of absence from Istituto di Fisica dell' Universitá, Milano, Italy. A. v. Humboldt fellow.

[1] For the case $n=3$ the result was communicated in [3]. Compare also Ref. [4] where, however, an unnecessary strong continuity condition was used.