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Causality in Non-Hausdorff Space-Times

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Abstract. Some general properties of completely separable, non-Hausdorff manifolds are studied and the notion of a non-Hausdorff space-time is introduced. It is shown that such a space-time must, under very general conditions, display a kind of causal anomaly.

1. Introduction

This paper is devoted to the study of a class of singularities exhibited by solutions of Einstein's equations. In the recent literature, several attempts of this sort appeared especially after the singularity theorems had been proved (e.g., in [1, 2]) stating that some pathological features must be present without much more information as to the extent of this pathology [3]. In some papers, such as [4, 5], various causal anomalies, geodesic incompleteness, anomalous sources, etc., in a concrete family of solutions are described. In other papers, one of these properties is investigated quite generally, for example the causal geodesic incompleteness [6] or incompleteness [7]. The problem to be solved here is to find general behaviour of those solutions whose maximal analytic extension is non-Hausdorff. At the first glance, these spaces seemed to be very strange containing bifurcate geodesics, curves with more than one endpoint, etc., [6]. Then, it has been shown that no such strangeness is present in a non-Hausdorff extension of the Taub-NUT space [8, 9]. Nevertheless, we shall see that all such space-times must be weakly acausal (Theorem 4).

2. Structure of Non-Hausdorff Manifolds

Non-Hausdorff manifolds are defined for instance in [10], p. 2. We consider here only a special case of them: the completely separable ones. First some symbolics. A non-Hausdorff, completely separable, n-dimensional, differentiable (C^k) manifold will be shortly said Y-manifold. In a Y-manifold W, there are at least two points p and q for which we find no two open disjoint sets A and B such that $p \in A$ and $q \in B$. This relation of points p and q will be written $p \curlyvee q$. If M and N are some subsets of W, then Y_M^N denotes the set of all points $x \in M$ for which there is $y \in N$