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On the Self-Adjointness of the $(g(x)\,\phi^4)_2$ Hamiltonian*

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Abstract. An alternate proof to that provided by Glimm and Jaffe of the essential self-adjointness of the Hamiltonian H for a relativistic scalar quantum field in two dimensional space-time with a "space cut-off" quartic interaction $H_I(g)$ is given. The proof depends mainly on the estimate $H_I^2(g) \leqq \text{const.}\,(N+I)^4$ and on the semiboundedness of $H = H_0 + H_I(g)$.

I. Introduction

We give an alternate proof of the essential self-adjointness of the total Hamiltonian $H = H_0 + H_I$ for a relativistic scalar quantum field in two-dimensional space-time with a "space cut-off" quartic interaction $H_I(g) = \int :\phi^4(x):g(x)\,dx$. This result has previously been established by Glimm and Jaffe using their singular perturbation theory [1] and a number of inequalities relating H, H_0, H_I and the number operator N [2].

II. Proof

We need the following information in our proof:

(a) Any vector ψ in the Fock Hilbert space $\mathscr{F}$ may be written $\psi = \sum_{n=0}^{\infty} \psi_{(n)}$ where the vector $\psi_{(n)}$ corresponds to an "n-particle state" (we will use the bracketed subscript exclusively to denote such vectors).

(b) H_0 is defined on a certain linear domain $\mathscr{D}(H_0) \subsetneqq \mathscr{F}$. The domain of H_I contains the space $\mathscr{D}'$ of all finite linear combinations of vectors $\psi_{(n)} \in \mathscr{F}$. The domain $\mathscr{D} = \mathscr{D}' \cap \mathscr{D}(H_0)$ is dense in $\mathscr{F}$ and H_0, H_I and H are symmetric operators on $\mathscr{D}$.

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