Commun. math. Phys. 21, 56—70 (1971)

Equilibrium States for a One Dimensional Lattice Gas

H. J. BRASCAMP

Institute for Theoretical Physics, Rijks-Universiteit, Groningen, The Netherlands

Received November 11, 1970

Abstract. The thermodynamic equilibrium state can be defined directly for an infinite system via an equilibrium condition or via the variational principle. Both definitions are used to calculate the equilibrium state for a one dimensional lattice gas with finite range interactions.

1. Introduction

In this paper, the equilibrium state for a one dimensional classical lattice gas with finite range interactions will be calculated explicitly. As a starting point, two different definitions for an equilibrium state for an infinite system will be taken. Usually, the equilibrium state for given temperature and chemical potential is found as the limit of finite volume Gibbs states. Because the limit may not be unique, it may be useful to define the notion of equilibrium state directly for the infinite system. There are two ways known.

1. *The Variational Principle.* An equilibrium state is a translationally invariant state, that maximizes the pressure [1]. By the occurrence of the mean entropy, translations play an essential role.

2. A number of equivalent *equilibrium conditions.* A set of equations is given, solutions of which are by definition equilibrium states. Examples are the KMS condition [2, 3], and the condition given by Dobrushin [5, 6] and Lanford and Ruelle [4]. For classical lattice systems, both conditions can be proved to be equivalent [7].

For the classical lattice gas, there is the following connection between the variational method and the equilibrium condition. If the interactions are translationally invariant, an invariant state satisfies the equilibrium condition precisely if it maximizes the pressure [4].

In the present paper, the above ideas will be applied to the one dimensional classical lattice gas with finite range, translationally invariant interactions. It is well known, that no phase transitions occur in this case [21].