

New “Coherent” States Associated with Non-Compact Groups*

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Abstract. Generalized “Coherent” States are the eigenstates of the lowering and raising operators of non-compact groups. In particular the discrete series of representations of $SO(2, 1)$ are studied in detail: the resolution of the identity and the connection with the Hilbert spaces of entire functions of growth $(1, 1)$. Also discussed are the application to the evaluation of matrix elements of finite group elements and the contraction to the usual coherent states.

I. Introduction

The definition and use of coherent states associated with the Heisenberg algebra is well known (Section II). The purpose of this paper is to generalize this notion to the Lie algebra of non compact groups. In particular, we deal with the simplest semi-simple Lie algebra of $SO(2, 1)$ isomorphic to the algebra of $SU(1, 1)$ and $SL(2, R)$. We call generalized “coherent” states the eigenstates of the ladder operators in the discrete series of representations. Generalizations of these continuous bases will be indicated. The new “coherent” states are useful mathematically, aside from their intrinsic interest, in the evaluation of matrix elements of the finite transformations of the group and will have physical applications as the ordinary coherent states have.

II. Coherent States Associated with the Heisenberg Algebra

In this Section we review briefly, for reference purposes, some important properties of the usual coherent states¹ which are introduced

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¹ For more details see, for example, Klauder, J. R.: Ann. Phys. (N. Y.) **11**, 123 (1960); Glauber, J.: Phys. Rev. **131**, 2766 (1963), and Klauder, J., Sudarshan, E. C. G.: Quantum Optics. New York: Benjamin 1968.