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Spherically Symmetric Similarity Solutions of the Einstein Field Equations for a Perfect Fluid*

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Abstract. Spherically symmetric space-times which admit a one parameter group of conformal transformations generated by a vector ξ^μ such that $\xi_{\mu;\nu} + \xi_{\nu;\mu} = 2g_{\mu\nu}$ are studied. It is shown that the metric coefficients of such space-times depend essentially on the single variable $z = r/t$ where r is a radial coordinate and t is the time. The Einstein field equations then reduce to ordinary differential equations. The solutions of these equations are analogous to the similarity solutions of the classical theory of hydrodynamics. In case the source of the field is a perfect fluid whose specific internal energy is a function of temperature alone, the solution of the field equations is uniquely determined by specifying data on the time-like hypersurface $z =$ constant and is a similarity solution. The problem of fitting a similarity solution to another solution of the field equations across a shock described by the hypersurface $z =$ constant is treated. A particular similarity solution for which $w = 3p$ obtains is shown to describe a Robertson-Walker space-time. This solution is fitted to a special static solution of the Einstein field equations which has a singularity at $r = 0$. The resulting solution of the Einstein field equations is shown to be regular everywhere except at $r = 0 \geqq t$ and the shock. The special Robertson-Walker metric is also fitted to a particular class of collapsing dust solutions (which are also similarity solutions) across a shock. The resulting solution is regular everywhere except at $r = t = 0$ and on the shock.

1. Introduction

In non-relativistic continuum mechanics there is a classical procedure for reducing the partial differential equations which characterize a given problem involving high symmetry to ordinary ones. This consists in assuming a specific form for the solution in which the dependent variables are taken to be essentially functions of a single independent variable. This variable is a dimensionless combination of the independent variables, namely the space coordinates and the time. Thus in a spherically symmetric problem where the independent variables are a distance from the center of symmetry, r, and the time t, the dependent variables are

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