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Commutators and Scattering Theory

I. Repulsive Interactions

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Abstract. We use commutators to find classes of operators which are smooth with respect to the Hamiltonian H for a system of quantum mechanical particles which repel each other. It follows that H is absolutely continuous, the wave operators are complete in many cases when they exist and limits of momentum observables as time approaches $\pm\infty$ exist even in cases where the long range of the interaction precludes existence of the wave operators.

1. Introduction

Let H_0 be the self adjoint operator in $\mathscr{K} = \mathscr{L}^2(\boldsymbol{R}^{3N})$ which represents the Hamiltonian for N free quantum mechanical particles, and H the Hamiltonian operator for the same system with repulsive interactions between pairs of particles and between each particle and a fixed center. Physically it is clear that if the potentials describing these interactions become small at large distances, for any initial state the interacting system should resemble the free system in the distant past and future, since no clustering of particles is possible. But even though the real complications of many particle scattering do not arise for purely repulsive interactions, the standard methods of scattering theory have failed to justify this physical certainty in some cases and do so only with difficulty in many others. The states φ which do appear free as $t\to\pm\infty$ in the sense that for some $\varphi_\pm$,

$$\|e^{-iHt}\varphi - e^{-iH_0t}\varphi_\pm\| \to 0 \quad \text{as} \quad t\to\pm\infty$$

are just those in the range of both wave operators

$$\Omega_\pm = \underset{t\to\pm\infty}{s\text{-lim}}\, e^{iHt}\, e^{-iH_0t}. \tag{1.1}$$

Thus what should be proved is that $\Omega_\pm$ exist and are complete in the sense that their ranges equal all of $\mathscr{K}$.

Problem 1. The wave operators can be shown to exist only if for all pair potentials $V: \boldsymbol{R}^3\to\boldsymbol{R}$, $V(x) = 0(|x|^{-1-\varepsilon})$ as $|x|\to\infty$, $(\varepsilon>0)$; thus the