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A New Approach to the Problem of Motion in General Relativity

ENRICO MASSA*
Istituto di Matematica dell'Università di Genova

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Abstract. The dynamical meaning of the equations $T^{ij}{}_{//j}=0$ is derived as a consequence of the mathematical structure of Einstein's equations. A generalization of Lichnerowicz's analysis of the gravitational equations is proposed.

§ 1. Introduction

In this paper we discuss the problem of motion for a material continuum in the framework of the general theory of relativity[1].
On this subject there is already a very extensive literature [1 ÷ 12]. The aim of the present contribution is to show that the general structure of the problem is intrinsically very simple.

To avoid unnecessary complications, we shall restrict our analysis to those systems whose four-velocity field V^i and density μ are expressed in terms of the energy-momentum tensor T_{ij} by the eigenvalue equation [13]

$$(T_{ij}+\mu g_{ij})\, V^j=0 \tag{1.1}$$

with the normalization

$$g_{ij} V^i V^j = -1\,. \tag{1.2}$$

We prove that if we assume the validity of Einstein's equations

$$G_{ij}=-kT_{ij} \tag{1.3}$$

and impose certain consistency conditions for the initial and boundary data of the gravitational problem, Eqs. (1.1), (1.2) are mathematically equivalent to the divergence equations

$$T^{ij}{}_{//j}=0\,. \tag{1.4}$$

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1 Throughout the paper, Latin indices will run from 1 to 4, and Greek indices from 1 to 3, unless otherwise stated. The metric is assumed to be of normal hyperbolic type with signature $(+\,+\,+\,-)$.

Partial derivatives will usually be indicated by a comma; covariant derivatives by a double vertical stroke (like in Eq. (1.4)).