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Unitary Equivalence of Fock Representations on the Weyl Algebra

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Abstract. A necessary and sufficient condition for unitary equivalence of pure quasi-free states over the Weyl algebra is proved. Some partial results on states over the Weyl algebra are formulated in Theorem 1, and Lemmas 1, 4, 5 and 6.

I. Introduction

As early as 1931 von Neumann [1] proved the uniqueness of the Schrödinger representation, for Boson-systems with a finite number of degrees of freedom. Afterwards a number of people [2] proved that for Boson-systems with an infinite number of degrees of freedom this theorem fails and that there exists a lot of inequivalent representations.

Kastler [3] gave for the first an algebraic formulation of this problem and proved von Neumann's theorem [1] in a more general form. He defined the underlying C^*-algebra for a free Boson-system roughly speaking generated by the Boson creation and annihilation operators, and formulated the problem of equivalence in terms of states on this algebra.

In this work we follow the same method and prove a necessary and sufficient condition (see Theorem 2 below) in order that two pure quasi-free states on the Boson C^*-algebra are unitarily equivalent.

The essential technical difficulty which we had to solve to derive the proof of the criterium, is the construction of finite symplectic subspaces of a sympletic space H which are invariant under two different complex structures on H. This problem is solved in the case that the product of the complex structures has a pure point spectrum.

Some other partial results on states over the Weyl algebra are formulated as remarks following the lemmas, the proofs being trivial extensions of the proofs of the lemmas.

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