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On the Nature of Turbulence

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Abstract. A mechanism for the generation of turbulence and related phenomena in dissipative systems is proposed.

§ 1. Introduction

If a physical system consisting of a viscous fluid (and rigid bodies) is not subjected to any external action, it will tend to a state of rest (equilibrium). We submit now the system to a steady action (pumping, heating, etc.) measured by a parameter μ[1]. When $\mu = 0$ the fluid is at rest. For $\mu > 0$ we obtain first a *steady state*, i.e., the physical parameters describing the fluid at any point (velocity, temperature, etc.) are constant in time, but the fluid is no longer in equilibrium. This steady situation prevails for small values of μ. When μ is increased various new phenomena occur; (a) the fluid motion may remain steady but change its symmetry pattern; (b) the fluid motion may become periodic in time; (c) for sufficiently large μ, the fluid motion becomes very complicated, irregular and chaotic, we have *turbulence*.

The physical phenomenon of turbulent fluid motion has received various mathematical interpretations. It has been argued by Leray [9] that it leads to a breakdown of the validity of the equations (Navier-Stokes) used to describe the system. While such a breakdown may happen we think that it does not necessarily accompany turbulence. Landau and Lifschitz [8] propose that the physical parameters x describing a fluid in turbulent motion are quasi-periodic functions of time:

$$x(t) = f(\omega_1 t, \ldots, \omega_k t)$$

where f has period 1 in each of its arguments separately and the frequences $\omega_1, \ldots, \omega_k$ are not rationally related[2]. It is expected that k becomes large for large μ, and that this leads to the complicated and irregular behaviour

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1 Depending upon the situation, μ will be the Reynolds number, Rayleigh number, etc.

2 This behaviour is actually found and discussed by E. Hopf in a model of turbulence [A mathematical example displaying features of turbulence. Commun. Pure Appl. Math. **1**, 303–322 (1948)].