Commun. math. Phys. 20, 143—166 (1971)


# On the Thermodynamic Limit of the B.C.S. State

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Received January 21, in revised form June 24, 1970

**Abstract.** We consider vector states in the Fock representation of the C.A.R. algebra, representing condensed pair states. We prove that in the thermodynamic limit these states give rise to a direct integral of gauge-dependent B.C.S. states.

## 1. Introduction

The algebra of the anticommutation relations (C.A.R. algebra) is an essential tool in the description of an infinite non relativistic Fermi system. A state on the C.A.R. algebra can be defined by giving the set of its $(n, m)$-point correlation functions $W_{n,m}$ (expectation values of Wick ordered monomials of fields operators). A simple class of states that has been extensively studied [1] ⋯ [3] is formed by the "quasi-free" states or "generalized" free states; a quasi-free state has the property that its truncated $(n, m)$ point functions $W^T_{n,m}$ vanish if $n+m>2$. One of the most important example of quasi-free state is provided by the B.C.S. state [4, 5].

The fact that for gauge-invariant quantities the state $\varrho$ over the C.A.R. algebra arising from the "Schafroth-condensed pair wave function" and the gauge-dependent B.C.S. state become equivalent in the thermodynamic limit, is well known by physicists since long time [6]; however, to the best of our knowledge, nobody has produced a rigorous proof of the identification of $\varrho$ with a direct integral of gauge dependent B.C.S. states. We shall produce here such a proof, at least for particular classes of "Cooper pairs".

However, the main reason for performing this work is to test some methods that might be useful when searching for physical states where the role of the "Cooper pairs" is played by "atoms" of more than two

* Partially supported by C.N.R.
** Equipe de Recherche associée au C.N.R.S. n° 154.