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About One Class of Representations of the Lie Algebra

M. Havlíček

Faculty of Mathematics and Physics, Charles University, Prague*

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Abstract. Let $R(G)$ be the skewsymmetric representation of the algebra G characterized by the following main property: if $G' \subset G$ is some subalgebra of G (possible noncompact) then $R(G')$ is integrable and reducible in the direct sum of irreducible representations of subalgebra G'.

The paper is devoted to the development of the elementary theory of the described representations, culminating in the proof of one version of Schur's lemma.

Introduction

The skewsymmetric representations of the Lie algebra are usually met in connection with unitary representations of the corresponding group. From this point of view they have been investigated also by Nelson [1] who found condition of the construction of the unitary representation $R(\tilde{G})$ of the simply connected group $\tilde{G}$ from skew-symmetric representation $R(G)$ of its Lie algebra G-the condition of the integrability of $R(G)$.

So we have, in the case of infinitedimensional skewsymmetric representations, two essentially different classes of representations: integrable and nonintegrable. Because assumption of the integrability $R(G)$ does not always have a physical meaning, the nonintegrable representations of the Lie algebra can be interesting from a physical point of view [2].

Also the use of the infinitedimensional Lie algebras in theoretical physics leads to the study of the representations of the Lie algebra without limiting on the integrable ones [3, 4]. In this case we do not usually have the corresponding group $\tilde{G}$ so that the concept of integrability of $R(G)$ loses its meaning.

Except for finitedimensional representations of the Lie algebras, the relatively simplest case arises for the semisimple ones, due to the results of Harish-Chandra [5]. If we denote $G' \subset G$ the maximal compact

* Myslíkova 7, Prague 1.