Commun. math. Phys. 20, 101—110 (1971)

On the Superpropagator of Fields with Exponential Coupling

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Received July 7, 1970

Abstract. We define the vacuum expectation value of the time-ordered product of two exponentials of free fields as a distribution using minimal singularity as a criterion. The implication of this definition for an exponentially self-coupled scalar field is studied in second order of a perturbation expansion.

I. Introduction

Lagrangians which involve an exponential of a spin zero field occur in a number of field-theoretic problems. Usually they result from the application of a formal canonical transformation to an originally polynomial Lagrangian. All exponential interactions are, of course, non-renormalizable according to the standard classification. However, a number of authors have tried to argue that for exponential couplings the definition of higher order perturbation theory terms need not suffer from the well-known ambiguities of non-renormalizable polynomial models. There is, in particular, an early paper by Okubo [1] and more recent publications by Volkov [2]. The essential idea is to expand only in powers of the interaction Lagrangian, leaving the exponential unexpanded.

In this paper we present a partial analysis of the structure of exponential interactions. For a discussion of the general term in a perturbation expansion it is necessary to give mathematical meaning to the following formal expression:

$$T\colon e^{f\phi(x_1)}\colon \ldots \colon e^{f\phi(x_n)}\colon = e^{if^2 \sum\limits_{i>j} \Delta_F(x_i - x_j)} \colon e^{f\phi(x_1)} \ldots e^{f\phi(x_n)}\colon \tag{1}$$

where ϕ is a free scalar field of mass m, f a constant. Our present work is restricted to the simplest non-trivial case, the definition of

$$iE_F(12) = \langle 0|\, T\colon(e^{f\phi(1)} - 1)\colon \colon(e^{f\phi(2)} - 1)\colon |0\rangle = e^{if^2 \Delta_F(12)} - 1 \tag{2}$$

which is often called the superpropagator for the exponential interaction. Without further restrictions, Eq. (2) contains an infinite number of arbitrary parameters, due to the singular structure of Δ_F. Previous work